

An Introduction to Computational Macroeconomics

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Bridging Languages and Scholarship

Series in Economic Methodology



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Preface

Dynamic General Equilibrium (DGE) models, either stochastic or deterministic, have become a fundamental tool in current macroeconomic analysis. Modern macroeconomic analysis is increasingly concerned with the construction, calibration and/or estimation, and simulation of Dynamic Stochastic General Equilibrium (DSGE) models. Notwithstanding the approach's important shortcomings, DSGE models are now in common use everywhere, spreading from academic research to Central Banks and other public and private economic institutions. This typology of models, which are based on the micro-foundation of the decisions taken by the different economic agents that are assumed to be rational, constitutes the reference theoretical framework of current macroeconomic analysis, and has two main characteristics. Firstly, these are dynamic models incorporating forward-looking variables, i.e., future variables in expected terms that are supposed to be determined in a context of rational expectations. Secondly, they are models in which the determination of macroeconomic variables is based on microeconomic analysis (microfoundation) and in a general equilibrium environment. In parallel, there is another typology of dynamic macroeconomic models, also in a context of rational expectations, which are non-micro-based, but are in turn based on the traditional IS-LM approach and the Phillips curve, or on more recent developments that incorporate monetary policy rules and a micro-based aggregate demand. Both types of models can be represented through a system of dynamic equations, which is composed of a set of equations that can be calculated numerically in a relatively simple way. However, apart from the greater or lesser complexity of this type of dynamic models at a theoretical level, the main problem that we find when teaching macroeconomics to undergraduate students or for those who want to get into this branch of macroeconomic modeling, is that they do not have an explicit solution, so they can only be solved numerically or by resorting to graphical analysis, mainly through phase diagrams, which is an important barrier to their incorporation into the teaching of advanced macroeconomics at the undergraduate level. This dynamic theoretical modeling approach makes the interrelation between current macroeconomics and numerical computing tools very tight, thus making it necessary to introduce a new approach that includes both elements in the learning of macroeconomics. Even the teaching of non-micro-based, simple dynamic models, requires the use of relatively complex analytical tools in order to work with systems of difference equations or differential equations. In all these cases, the usual strategy at introductory levels is to apply graphical analysis, through the construction of the phase diagram, as a basic instrument for

carrying out macroeconomic analysis. However, even the use of the phase diagram assumes a certain level of difficulty while also presenting a number of limitations. All these elements result that the macroeconomic analysis scheme that is used by the majority of economic authorities, from governments to central banks, international organizations, and research centers, is not known by economics graduates, which is very striking, as the teaching of macroeconomics in universities at an undergraduate level is one step behind its use at academic level and its practical application in the real world.

However, it is feasible to introduce dynamic macroeconomic models in a relatively understandable way, as well as their numerical solution at a basic level. In fact, the contents of this book are based on the idea of introducing a new approach to the teaching of macroeconomics, which allows, through techniques that are relatively easy to implement and comprehensible at introductory levels, the numerical solution of these models and their use for macroeconomic analysis. Although current macroeconomic analysis need knowledge of some complex computer tools and mathematical techniques, in practice, these models can be solved with simple computer software as it is the case of a spreadsheet. A spreadsheet contains a set of tools that can be used for the numerical solution of any type of dynamic macroeconomic models. The proposed approach offers a new perspective for the teaching of macroeconomics at an undergraduate level, since it allows the flexible use of dynamic models for its application to macroeconomic analysis, being at the same time an introduction to numerical solution methods. All this can be done with accessible computational software that is easy to learn and is already well known, at least to some degree, by undergraduate students. This is the strategy pursued in this book.

As we have indicated previously, in general terms, we can distinguish two types of models used in current macroeconomic analysis. Firstly, simple dynamic models that do not have a microeconomic foundation, but that use rational expectations. An example of these models is the overreaction or overshooting of the exchange rate, or the IS-LM traditional Phillips Curve model. Secondly, the models based on the so-called Dynamic General Equilibrium model, both stochastic and deterministic, or Ramsey's optimal growth model, which constitute the theoretical framework of reference used in current macroeconomic analysis. These macroeconomic models do not have, in general, a closed-form solution and, therefore, can only be solved using numerical methods. This makes computational macroeconomics a key element in current macroeconomic analysis and the introduction of a necessary change in the way in which students are introduced to macroeconomic analysis.

The objective of this book is to make an introduction, at the most basic level, to computational macroeconomics and to the resolution of micro-based

macroeconomic dynamic general equilibrium models. The fundamental idea is to show that it is relatively achievable task to solve a dynamic macroeconomic model, whether micro-based or not, using tools that are available to everyone, without the need to learn computer programs and languages of a certain complexity, which limit their practical application. This book includes a series of computer exercises with alternative macroeconomic models, divided into three parts: Basic Dynamic Models, Introduction to Dynamic General Equilibrium, and Economic Growth. The objective is to solve different and simple models by using numerical and computational methods. Computation in this context means we will use a computer to solve these models and that their solution will be done numerically. All the models that we study are solved and computed in a deterministic environment, although their resolution in a stochastic environment does not pose major problems since a spreadsheet contains tools for the incorporation of random variables. However, we consider that the deterministic environment is much more illustrative for learning purposes than a stochastic environment, so we have not considered it relevant to introduce the stochastic component in the studied models.

The first part of the book includes a series of numerical exercises using simple linear dynamic models. In Chapter 1, an exercise of a general model, not representing any specific macroeconomic model, is carried out, based on a dynamical system composed of two difference equations. The objective of this exercise is to introduce the fundamental concepts to be use next, as well as to present the analytical framework to be use for the numerical solution of these types of models. Next, in Chapter 2, we present a dynamic version of the IS-LM model. The IS-LM model constitutes the basic theoretical framework used in the teaching of macroeconomics at intermediate levels. The model considered here is a dynamic version, where there is a process of adjustment in the price level over time (the Phillips curve). To close this first part, Chapter 3 presents Dornbusch's overshooting model of the exchange rate. This is a model for a small open economy that allows explaining the response of the nominal exchange rate to different shocks.

The second part of the book is composed of five chapters. In Chapter 4, we present the basic household problem in terms of the consumption-savings decision. This analysis constitutes a fundamental piece in the dynamic models of general equilibrium that are used in the current macroeconomic analysis. Chapter 5 introduces the consumption-leisure decision in the household problem, including leisure as an additional argument in the utility function, from which the labor supply is derived. Chapter 6 analyzes the consumption-savings decisions by households, but also considering the role of the government, with the objective of studying how households' decisions change according to tax rates and social security contributions. Chapter 7

studies the investment decision through Tobin's Q model. To close this second part, Chapter 8 presents a very simple version of the dynamic general equilibrium model.

The last part of the book focuses on the analysis of economic growth models. We study two different models. First, in Chapter 9, we conduct a numerical solution of Solow's exogenous growth model, which is the basic framework for studying the long-term behavior of an economy. Finally, in Chapter 10, we present Ramsey's optimal growth model. In this case, the analysis is similar to the dynamic general equilibrium model presented above, but considering population growth.

The software used in this book is Excel[®] from Microsoft[®]. Excel is a spreadsheet, similar to others such as Open Office Calc. The advantages of spreadsheets are that they are easy to handle, widely used and known by a large number of users, while having a high capacity to perform a wide variety of numerical operations. Barreto (2015) presents a series of arguments about why Excel is an adequate tool for the teaching of macroeconomics, indicating that spreadsheets have important advantages when communicating ideas and presenting statistical data to students.¹ In fact, spreadsheets are a resource that is increasingly used in the teaching of economics, Strulik (2004)², and Gilbert and Oladi (2011)³ being examples. More recently, Barreto (2016) proposes a set of spreadsheets for the teaching of macroeconomics at introductory and intermediate levels⁴. Another macroeconomics manual with simulations in Excel is that of Carlin and Soskice (2015)⁵. A book with different examples of computational economy where other computer programs are also used, apart from spreadsheets, is Kendrick, Mercado and Amman (2006).⁶

This book shows that the numerical solution of dynamic general equilibrium models is a relatively simple task that can be done in a spreadsheet. Micro-

¹ Barreto, H. (2015). Why Excel?, *Journal of Economic Education*, 46(3), 300-309.

² Strulik, H. (2004). Solving Rational Expectations Models Using Excel, *Journal of Economic Education*, 35(3): 269-283.

³ Gilbert, J. y Oladi, R. (2011). Excel Models for International Trade Theory and Policy: An Online Resource, *Journal of Economic Education*, 42(1): 95.

⁴ Barreto, H. (2016). *Teaching Macroeconomics with Microsoft Excel*. Cambridge University Press.

⁵ Carlin, W. y Soskice, D. (2015). *Macroeconomics: Institutions, Instability, and the Financial System*. Oxford University Press.

⁶ Kendrick, D. M., Mercado, P. R. y Amman, H. M. (2006). *Computational Economics*. Princeton University Press.

based macroeconomic models can be solved by following two alternative approaches. First, they can be solved using Excel's Solver tool, which allows the maximization of an objective function subject to certain restrictions. Secondly, these models can be solved directly using a linear approximation to the non-linear equations of the model in terms of the deviations from the steady state. This allows us to calculate in a simple way the eigenvalues associated with the system and use them to calculate numerically the readjustment in the forward-looking variables facing a disturbance when the solution is a saddle-point, in a similar way to the procedure that we would use for solving more traditional non-micro-based dynamic models.

Finally, the book includes a series of appendices with some basic mathematical concepts that are necessary for the performed analysis. These appendices also include a series of computer code, where the different models are solved in both MATLAB® and DYNARE, which are computational tools commonly used in current macroeconomic analysis. For those who want to use their smartphone or tablet for the solution of stochastic dynamic general equilibrium models, see Blake (2012).⁷

Anelí, Trinidad and José Luis
Málaga, December 2019

⁷ Blake, A. P. (2012). DSGE modeling on iPhone/iPad using SpaceTime. *Computational Economics*, 40(4): 313-332.

Part I: Basic Dynamic Systems

1. An introduction to computational dynamic systems

1.1. Introduction

Dynamic macroeconomic model can be written as a system of dynamic equations: a system of differential equations if the model is defined in continuous time or a system of difference equations if we define it in discrete time. The used temporal approach depends on the type of analysis to be carried out. At a theoretical level, it is usual to develop macroeconomic models in continuous time, which allows the application of certain analytical instruments, such as the phase diagram, in order to obtain a graphical representation of the model to be used for shock analysis. However, if our objective is the numerical resolution of the model, then the approach to be used would be the discrete-time approach. In this case, the macroeconomic model is defined as a system of difference equations. In any case, both specifications are equivalent and lead to similar results, although the exact specification of each equation may be different in both temporal contexts. The set of equations that make up a model includes three elements: endogenous variables (unknowns), exogenous variables and parameters. Given some values for the parameters, which have to be calibrated or estimated, and given some values for the exogenous variables, which are supposed to be given, we can numerically calculate the corresponding value of the endogenous variables. On the other hand, the numerical resolution requires the use of specific functional forms that determine the relationship between these elements, so it is necessary to have a specific functional form for each equation.

In this first chapter, we carry out a simple computational exercise with the aim of illustrating how a computer can be used to perform numerical simulations of dynamical models. For this, we will use a relatively easy-to-use tool widely known as a spreadsheet. In particular, we will use the Microsoft Office Excel spreadsheet, although we could also use other similar spreadsheets, such as Open Office Calc. In various appendices, we include similar exercises but using other tools commonly used in macroeconomic analysis, such as MATLAB, or based in a software designed specifically for the resolution of dynamic macroeconomic models with rational expectations, such as DYNARE. The fundamental idea of this first chapter is not to solve a particular macroeconomic model but to show how to solve and numerically

simulate a dynamic system in discrete time and show that it is a relatively simple task.

Solving this type of dynamic systems numerically has important advantages when carrying out different analyses to study the effects of different disturbances on the economy. In the first place, it allows the obtaining of the temporary paths of the endogenous variables in response to a shock, that is, it allows one to know how each macroeconomic variable reacts to a disturbance represented by a change in the exogenous variables. This is what is known as impulse-response function or transition dynamics, an instrument widely used in macroeconomic analysis. For that, it is enough to make a plot of the value of each variable as a function of time to appreciate its dynamic behavior in response to a change in the economic environment. Secondly, it allows one to carry out sensitivity experiments, studying the behavior of the model according to the value of the different parameters. Thus, once the model is solved and computed numerically, it is only necessary to change the value of a parameter to observe how the economy reacts, both in terms of its equilibrium and its dynamics. This means that we can simulate the different trajectories of the endogenous variables in response to a change in the exogenous variables, conditioned to a given value of the parameters of the model.

In this first chapter, we numerically simulate Richardson's model of arms race. This is a model composed of two linear difference equations that represent the behavior over time of two countries, potentially enemies, and their decisions on the stock of weapons of each of them in relation to the stock of weapons of the other country. The choice of this particular model as the starting point obeys several factors. First, it is a very popular model in both political science and economics (used, for example, in microeconomics to analyze the behavior of a duopoly in a dynamic context where the endogenous variables are the market shares of the firms). Secondly, it is a model with a very simple structure, so it is easy to understand. Finally, it is a model in which, depending on the value of the parameters, we obtain the two types of solutions we can obtain in a macroeconomic model: global stability (when all trajectories bring us to equilibrium) or saddle point (when only a few trajectories take us to the steady state, while the rest take us away from it). In Section 2, we present a brief description of the difference equations as well as the notation of the dynamic system that we will solve in discrete time. In Section 3, we present an example of a dynamic system, and how to simulate it numerically. Section 4 shows the numerical resolution of the dynamic system in Excel, describing all the elements that must be entered in the spreadsheet to obtain the numerical solution. Section 5 uses the previously constructed spreadsheet to analyze the effects of a given disturbance, that is, a change in the exogenous variables. Finally, in Section 6, a similar analysis is carried out

but in terms of a change in the value of the parameters. This is what is called sensitivity analysis. In the particular case of the model that we are analyzing, the sensitivity analysis has special relevance, since the system stability can change from a solution with global stability to a saddle point solution.

1.2. A simple dynamic model

In general, we can write a macroeconomic model as a dynamic system that is composed of a set of difference equations:

$$\Delta \mathbf{X}_t = F(\mathbf{X}_t, \mathbf{Z}_t) \quad (1.1)$$

where \mathbf{X}_t is a vector of endogenous variables of dimension n , \mathbf{Z}_t is a vector of exogenous variables of dimension m , and $F(\cdot)$ is a particular mathematical function. The symbol Δ indicates variation with respect to time, such that:

$$\Delta \mathbf{X}_t = \mathbf{X}_{t+1} - \mathbf{X}_t \quad (1.2)$$

that is, the variation at the moment t is defined as the difference between the value of the variable at the moment $t + 1$ and its value in the previous period, t .¹ In this way, the value of the endogenous variables at the moment t is equal to their value in the previous period plus their rate of variation during that period, so we could define:

$$\mathbf{X}_t = \mathbf{X}_{t-1} + \Delta \mathbf{X}_{t-1} \quad (1.3)$$

Expression (1.1) indicates that the variation with respect to the time of the vector of endogenous variables is a function of the value of these endogenous variables and the value of the exogenous variables. In this way, what this system of equations indicates is how the endogenous variables move along time as a function of the changes that occur in the exogenous variables.²

¹ Another way also used to define the variations of the variables at the moment t is:

$$\Delta \mathbf{X}_t = \mathbf{X}_t - \mathbf{X}_{t-1}$$

The use of one specification or another depends on whether the value of the variable is defined at the beginning of the period or at the end of it. Both specifications are not equivalent, resulting in different results.

² Alternatively, the system of equations can be written as:

$$\mathbf{X}_t = G(\mathbf{X}_t, \mathbf{Z}_t)$$

that is known as normal form (see Appendix A). However, to make the analysis equivalent to the one carried out in continuous time, in which the equations are specified in terms of the derivative with respect to the time of the endogenous variables, which is the form usually adopted in macroeconomic analysis, we have opted to carry out all the analyzes with the representation given by the specification (1.1).

As a previous step, we will proceed to apply logarithms to all variables (except those that are defined in percentage terms, such as the interest rate), denoted by a lowercase letter. Therefore, we define:

$$x_t = \ln X_t \quad (1.4)$$

Hence, the growth rate for each variable, can be defined as:

$$\begin{aligned} \Delta x_t &= x_{t+1} - x_t = \ln X_{t+1} - \ln X_t = \ln \left(\frac{X_{t+1}}{X_t} \right) \\ &= \ln \left(1 + \frac{X_{t+1} - X_t}{X_t} \right) \cong \frac{X_{t+1} - X_t}{X_t} \end{aligned} \quad (1.5)$$

that is, the variation with respect to time in logarithmic terms is approximately equivalent to the growth rate of the variables, which is very useful, since in most cases, our interest will focus on knowing how the growth rate of macroeconomic variables is. Indeed, the rate of growth of the output level is named economic growth, the rate of growth of prices is named inflation, etc.

Initially, we will work with systems of linear difference equations. In practice, many of the equations that make up a macroeconomic model, mainly micro-based macroeconomic models, are non-linear equations. We can suppose that the system of difference equations is linear or construct a linear approximation of the same in the case that the initial equations are non-linear and write it as:

$$\Delta \mathbf{x}_t = A \mathbf{x}_t + B \mathbf{z}_t \quad (1.6)$$

where A is a matrix $n \times n$, B is a matrix $n \times m$ and \mathbf{z}_t is the vector of exogenous variables $m \times 1$. Although we specify the vector of exogenous variables as a function of time, in practice it is a constant, except when an exogenously determined disturbance that affects its value occurs. The fact of defining the exogenous variables as a function of time is only intended to differentiate them from the parameters of the model, which are also constant.

For illustrative purposes, we assume that $n = 2$, that is, our model would be composed of two difference equations and two endogenous variables. The analysis would be similar if the number of equations in the system is greater. In particular, the typical dynamic system with which we will work is the following:

$$\begin{bmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \end{bmatrix} = A \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + B \mathbf{z}_t \quad (1.7)$$

being

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (1.8)$$

and where the dimension of the matrix B will depend on the number of exogenous variables included in the vector \mathbf{z}_t .

1.3. An example of dynamic system: Richardson's model of arms race

Before proceeding with the numerical resolution of different macroeconomic models, we begin by solving a general dynamic system that does not correspond to any specific macroeconomic model. This is the arms race model, developed by Lewis Fry Richardson (1881-1953) in 1919.³ This model represents two countries, which are potentially enemies, and where the endogenous variable is the armament stock of each of them. This simple setup is very intuitive to understand the dynamic behavior of the endogenous variables. Depending on the value of the parameters, the solution of the model is determined by the existence of global stability or by a saddle point, two different solutions that we can find in macroeconomic models to be study later.

Richardson's model of arms race can be defined through the following linear dynamic system, in which we have two endogenous variables ($x_{1,t}, x_{2,t}$) and two exogenous variables ($z_{1,t}, z_{2,t}$), where the stock of armament of each country depends positively on the stock of armament of the other country and negative on our own stock of armament. In matrix notation:

$$\begin{bmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \end{bmatrix} = \underbrace{\begin{bmatrix} -\alpha & \beta \\ \gamma & -\delta \end{bmatrix}}_A \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \underbrace{\begin{bmatrix} \theta & 0 \\ 0 & \eta \end{bmatrix}}_B \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} \quad (1.9)$$

where the matrix A represents the matrix of coefficients associated with the endogenous variables and the matrix B the coefficients associated with the exogenous variables. The endogenous variables represent the armament stock of each country (countries 1 and 2), while the exogenous variables represent any factor that affects the decision on the armament stock in each of the countries. All parameters, represented by Greek letters, are defined in positive terms.

Therefore, the equations that define the behavior over time of the two endogenous variables are the following:

³ Richardson, L. F. (1919). The mathematical psychology of war. W. Hunt: Oxford.
 Richardson, L. F. (1935a). Mathematical psychology of war. *Nature*, 135: 830-831.
 Richardson, L. F. (1935b). Mathematical psychology of war. Letter to the Editor. *Nature*, 136: 1025.

$$\Delta x_{1,t} = -\alpha x_{1,t} + \beta x_{2,t} + \theta z_{1,t} \quad (1.10)$$

$$\Delta x_{2,t} = \gamma x_{1,t} - \delta x_{2,t} + \eta z_{2,t} \quad (1.11)$$

This model describes the reaction functions of each country with respect to the decision on armament of the other. Thus, observing the sign of the parameters of the matrix A , we find that the dynamics of the armament stock in a country depends negatively on its own armament stock and positively on the armament stock of the other country. The parameter β represents how country 1 reacts to the stock of weapons in country 2, while the parameter γ indicates how country 2 reacts to the stock of weapons in country 1.

Structure of the model of arms race	
Reaction function of country 1	$\Delta x_{1,t} = -\alpha x_{1,t} + \beta x_{2,t} + \theta z_{1,t}$
Reaction function of country 2	$\Delta x_{2,t} = \gamma x_{1,t} - \delta x_{2,t} + \eta z_{2,t}$
Variation of the armament stock country 1	$\Delta x_{1,t} = x_{1,t+1} - x_{1,t}$
Variation of the armament stock country 2	$\Delta x_{2,t} = x_{2,t+1} - x_{2,t}$

Next, we will study two concepts that are fundamental for the analysis to be done later: first, the concept of equilibrium, which we define as the steady state, and second, the concept of stability of the system, which will indicate how the trajectories of the variables are with respect to the steady state.

1.3.1 Definition of equilibrium: The Steady State

The concept of equilibrium used in dynamic analysis is the so-called Steady State. While in a static analysis the concept of equilibrium typically relates to a situation in which the supply of a certain variable is equal to its demand (market clearing), in the dynamic analysis, the concept of equilibrium relates to a situation in which economic variables remain constant over time (their time variation is zero, so they take the same value period by period). The value of the variables in steady-state, denoted by a horizontal bar on the variable, therefore, would be defined by that situation in which the variation with respect to the time is equal to zero.

$$\bar{x} \Rightarrow \Delta x_t = f(x_t, z_t) = 0 \Rightarrow x_t = \bar{x} \quad (1.12)$$

This means that the steady state of a dynamic system is given by a vector of n -dimension zeros, for each of the difference equations of the system presenting the temporal variation of the endogenous variables ($\Delta x_t = 0$).

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